Evaluation of Two Different $k-\varepsilon-\overline{vv}-f$ Turbulence Models for Natural Convection in a Rectangular Cavity

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Abstract

A numerical study of natural convection in a rectangular cavity with the $k-\varepsilon-\overline{vv}-f$ models is presented. The primary emphasis of the study is placed on the investigation of accuracy and numerical stability of the $k-\varepsilon-\overline{vv}-f$ models for a natural convection problem. Both the original $\overline{vv}-f$ model [1] and its modified one [2] are considered. Both models are applied to the prediction of natural convection in a rectangular cavity together with the two-layer model [3]. The original model exhibits the numerical stiffness problem when used with the segregate solution procedure like the SIMPLE algorithm and a simple remedy for this problem is proposed. The computed results are compared with the experimental data commonly used for validation of the turbulence models. It is shown that the original $\overline{vv}-f$ model predicts accurately the mean velocity, velocity fluctuation, Reynolds shear stress, turbulent heat flux and the local Nusselt number at the hot wall. The modified $\overline{vv}-f$ model predicts well all the quantities, but the accuracy of solution is a little deteriorated than that of the original model. The two-layer model predicts poorly the mean vertical velocity component and underpredicts the turbulent quantities. As is already known in the literature, the modified $\overline{vv}-f$ model enhances greatly the numerical stiffness problem of the original model..

Keyword: *$k-\varepsilon-\overline{vv}-f$ turbulence models, turbulent natural convection*

1. Introduction

Accurate prediction of natural convection is very important for investigating the fluid flow and heat transfer in a reactor vessel auxiliary cooling system adopted in the Korea advanced liquid metal reactor design. The natural convection also plays a very important role in the thermal stratification in the upper plenum of liquid metal reactor during the scram condition. Despite its importance in practical engineering problems, the turbulent natural convection has received attention only from a few researchers. One of the difficulties of computation of natural convection by the conventional $k-\varepsilon$ model is the validity of the wall function method, which is based on the local equilibrium logarithmic velocity and temperature assumptions. The logarithmic wall functions were originally derived for forced-convection flows and do not hold for natural convection boundary layers. Due to this problem, most of the previous authors used the low-Reynolds number turbulence models for computation of natural convection problems, however, a limited success is reported. The use of second moment closure may result in better solutions, however, the second moment modeling of natural convection requires modeling of various terms in the transport equations for the turbulent heat flux vector, the temperature variance and the dissipation of temperature variance, and its use in the practical engineering problems is still questionable due to its complexity and demand of high computer resources. Kenjeres and Hanjalic [4] and Kenjeres [5] developed an algebraic flux model (AFM hereafter) together with the low-Reynolds number turbulence model and applied it to the prediction of various natural convection problems. Satisfactory results are obtained when their results are compared

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with the experimental data and LES results. A good feature of AFM developed by Kenjeres and Hanjalic [4] is its simplicity, it requires only one additional solution of the transport equation for temperature variance. This model will be used for computation of natural convection in a rectangular cavity together with the $k-\varepsilon-\overline{vv}-f$ model developed by Durbin [1] in the present study. Durbin [1] developed a $k-\varepsilon-\overline{vv}-f$ model around the elliptic relaxation method for representing the near wall phenomenon. Unlike the low-Reynolds number turbulence models, this model is done without the aid of wall damping functions and it has been successfully applied to separated flows [1] and other flows. The original $\overline{vv}-f$ model works well with the coupled, implicit scheme, but it is known that this model exhibits a numerical stiffness problem when used with the segregate solution procedure like the SIMPLE algorithm. Lien and .Kalitzin [2] developed a modified model (n=6 model) to avoid this problem. In the present study a simple way of avoiding the numerical stiffness problem for the original $\overline{vv}-f$ model is proposed. The primary objective of the present study is evaluation of the $k-\varepsilon-\overline{vv}-f$ model with the algebraic flux model for the natural convection problem. The relative performances between the original model and the modified model are investigated

2. Numerical Method

The turbulence models considered in the present study are implemented in the computer code specially designed for evaluation of turbulence models. The computer code employs the nonstaggered grid arrangement and the SIMPLE algorithm for pressure-velocity coupling. The higher order bounded HLPA scheme is used for treating the convection terms. Calculations are performed using the 82×122 numerical grids. The computations are continued until the maximum residual of all computed variables is less than 10^{-6}. This convergence criterion is sufficiently small to assure the convergence.

3. The Test Problem

The test problem considered in the present study is a natural convection of air in a rectangular cavity with aspect ratio of 5:1. The height of cavity is $H=2.5m$ and the width of cavity is $L=0.5m$ and the temperature difference between hot and cold wall is 45.8K. The Rayleigh number based on the height of cavity is $Ra=4.5\times10^{10}$ and Prandtl number is $\Pr=0.7$. King [6] has made extensive measurements for this problem and experimental data are reported in Cheesewright et al. [7] and King [6].

4. Results and Discussion

Calculations are performed using the 82×122 numerical grids. The computations by the two-layer model [3] were stable, but the solutions were a little affected by the location of the interfaces between the two regions where the $k-\varepsilon$ model is used and the one equation model is used. We gradually changed the locations of the interfaces until the solution is no longer changed. We experienced the numerical stiffness problem when we used the original $\overline{vv}-f$ model. The numerical stiffness problem occurred near the boundary, and we developed a simple technique at the wall boundary to avoid such a problem. We also found that the initial conditions also affect the numerical stability. The results of the two-layer model are used for the initial conditions for the $\overline{vv}-f$ model computations.

Figure 1 shows the streamlines and isotherms predicted by the two-layer model and the two different $\overline{vv}-f$ models. There exist only weak interactions between the two boundary layers near the hot and cold walls and a rotating core. The width of the cavity is large enough to establish separate boundary layers at hot and cold walls and the core of the cavity is stratified. The isotherms predicted by the $\overline{vv}-f$ models are equally spaced, while those by the two-layer model are not equally spaced, indicating that the vertical centerline temperature distribution is not linear. The vertical variation of the temperature at the center region is smallest in the two-layer computation and is largest in the original $\overline{vv}-f$ model computation. There exists a small gradient of the temperature across the

horizontal direction at the center region in the predictions of the two-layer, while the temperature distributions at the center region are nearly flat in the prediction of the $\overline{vv}-f$ models.

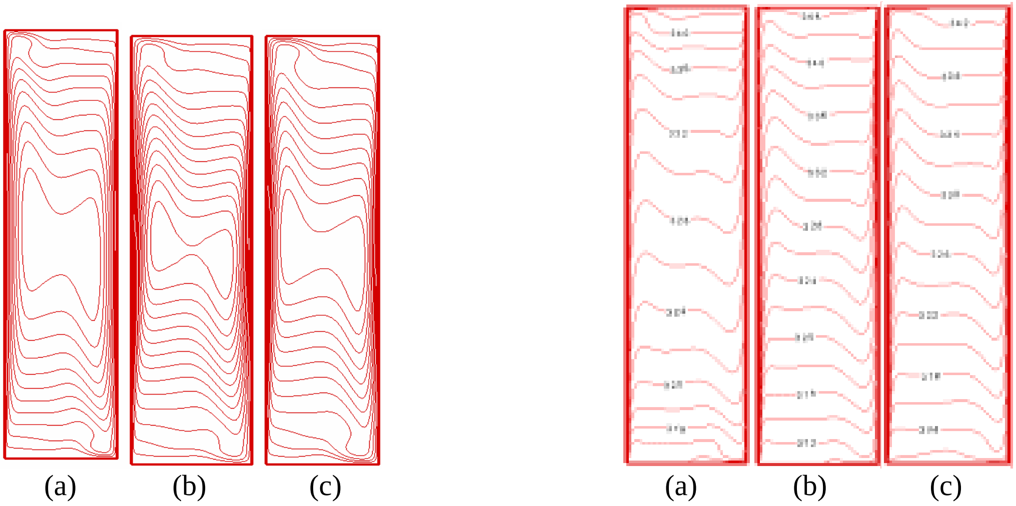


Fig.1 The predicted streamlines and isothermal lines : (a) Two-layer model, (b) $\overline{vv}-f$ model (n=1), (c) $\overline{vv}-f$ model (n=6)

We first compare the predicted results with the measured data reported in Cheesewright et al. [7] for the vertical mean velocity and the vertical velocity fluctuation at the mid-height (y/H=0.5) of the cavity. Figure 2 show the comparisons of the predicted results with measured data for the vertical velocity component at y/H=0.5. As shown in the figures, the agreement between the measured data and the predictions by the two $\overline{vv}-f$ models are very good, while the two-layer model poorly predicts it. The two-layer model produces a laminar-like solution for the vertical velocity component in the near wall region. It is noted that the experimental data for the vertical velocity component show a small deviation of symmetry, indicating that there is a three-dimensional effect.. Figure 3 shows the comparison of the predicted vertical velocity fluctuation at the mid-height (y/H=0.5) with the experimental data. It is shown that the modified $\overline{vv}-f$ model predicts best the vertical velocity fluctuation when compared with the measured data, and the original $\overline{vv}-f$ model slightly underrpredicts it in the near wall region. The two-layer model severely underpredicts it in the near wall region. All the models predict almost the same level of vertical velocity fluctuation in the center region.

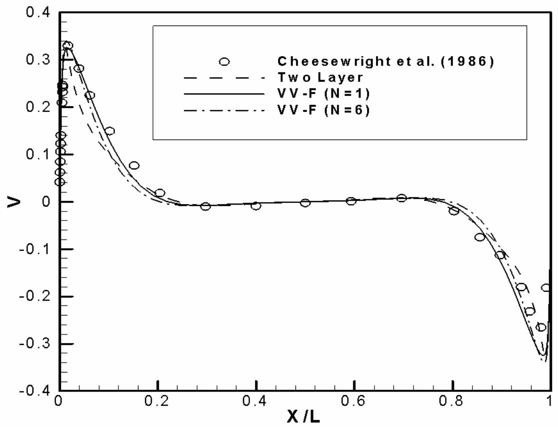


Fig.2 Mean vertical velocity profiles at y/h=0.5

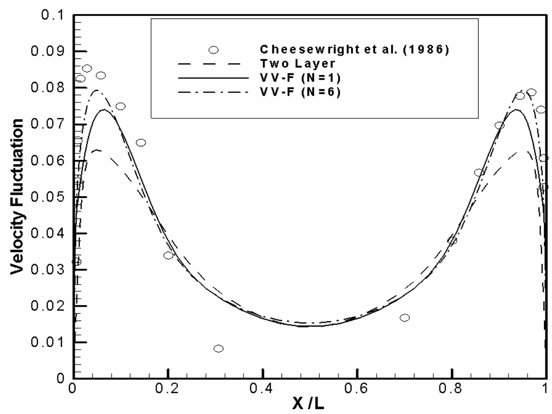


Fig.3 Vertical velocity fluctuation profiles at y/H=0.5

Figure 4 shows the comparison of the predicted vertical centerline temperature profiles at x/L=0.5 with the measured data by King [6]. We first note that the measured data of the vertical centerline temperature do not show the linear variation, and Cheesewright et al. [7] explain this phenomenon is due to the insufficient insulation of the side and upper walls. The heat loss from the side and upper walls causes the reduction of temperature, and the centerline temperature deviates from the linear variation at the upper region of the cavity. The predicted results by the $\overline{vv}-f$ models clearly exhibit the linear variation while the prediction by the two-layer model does not show such a trend. The differences between measured data with the predictions by the $\overline{vv}-f$ models are believed to be due to the insufficient insulation of the side and upper walls, however, the prediction by the $\overline{vv}-f$ models at the lower region of the cavity agrees well with the measured data. Figure 5 shows the profiles of the predicted Reynolds shear stress $\overline{uv}$ at the mid-plane (y/H=0.5) of the cavity together with the measured data. The predicted results show the symmetry, while the experimental data show asymmetry and it is probably due to the heat loss at the side and upper walls. The original $\overline{vv}-f$ model predicts best the $\overline{uv}$ profile near the hot wall, however, the predicted result does not agree well with the measure data near the cold wall. The magnitude of $\overline{uv}$ predicted by the two-layer model is smaller than that by the original $\overline{vv}-f$ models. The prediction by the modified $\overline{vv}-f$ model is nearly the same as that of the two-layer model. Throughout the present investigation it is observed that the prediction by the two-layer model agrees well with the measured data near the cold wall, while the predictions by the $\overline{vv}-f$ models agree better with experimental data near the hot wall.

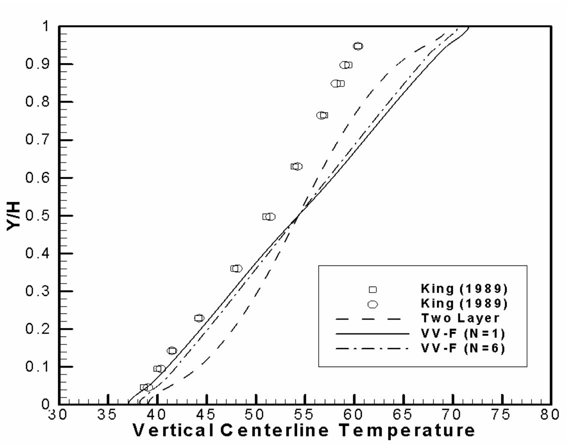


Fig.4 Vertical centerline temperature profiles at y/H=0.5.

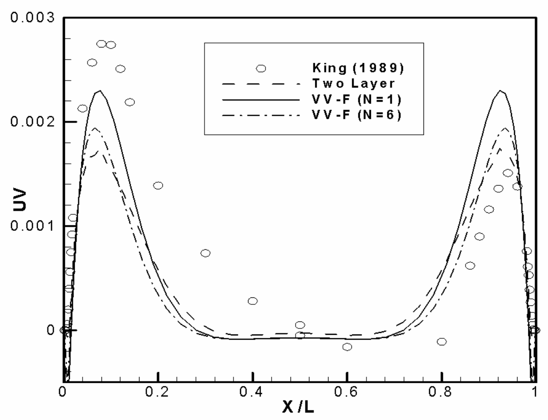
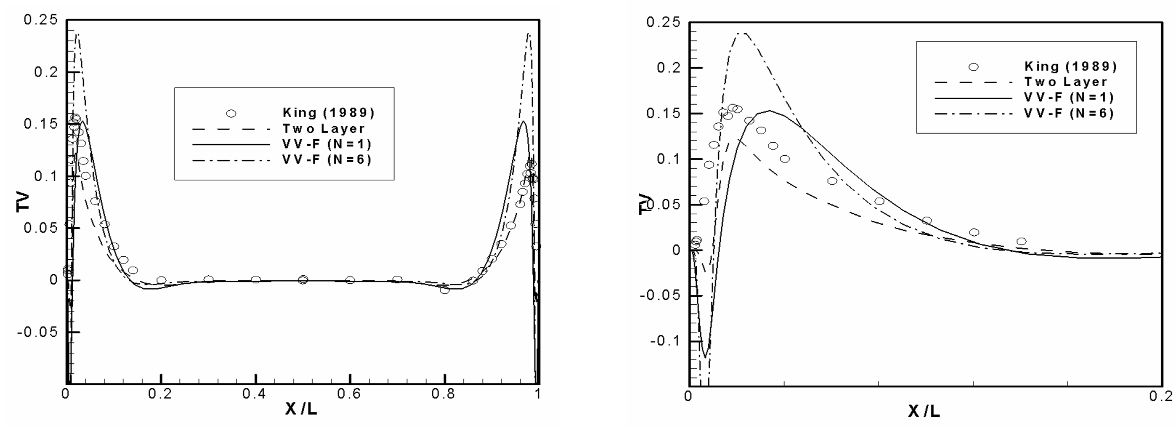


Fig.5 Turbulent shear stress $\overline{uv}$ profiles at x/L=0.5

Figures 6 shows the profile of the predicted vertical turbulent heat flux $\overline{\theta v}$ at the mid-plane (y/H=0.5) of the cavity with the measured data. It is noted that the vertical turbulent heat flux vector $\overline{\theta v}$ plays the most important role in the dynamics of turbulent kinetic energy in the buoyant turbulent flows and influences directly the overall prediction of all quantities. The AFM used in the present study contains all temperature and mean velocity gradients together with the correlation between the gravity vector and temperature variance. The level of the predicted turbulent heat flux $\overline{\theta v}$ near the hot wall by the original $\overline{vv}-f$ model agrees well with the measured data, however, it is skewed a little toward the center region as shown in Fig. 6. The predicted result by the modified $\overline{vv}-f$ model generally follows the trend of the measured data, however, it over-predicts the vertical turbulent heat flux in the region close to the wall. The two-layer model under-predicts the vertical turbulent heat flux near the hot wall region, but the prediction near the cold wall agrees well with the measured data.



(a) overall view (b) partial view near the hot wall

Fig.6 Vertical turbulent heat flux $\overline{\theta v}$ profiles at y/H=0.5

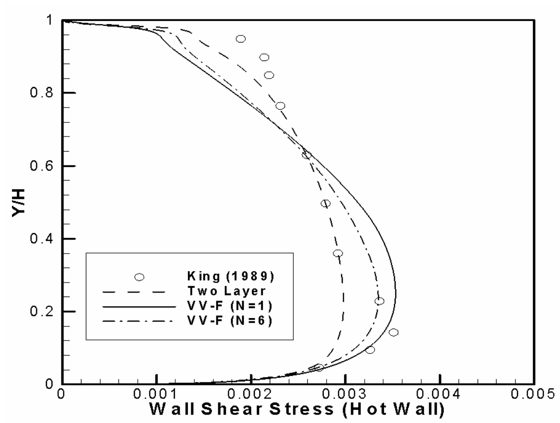


Fig.7 Wall shear stress distributions

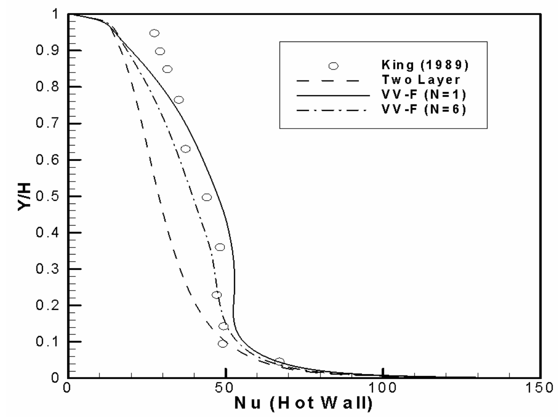


Fig.8 Local Nusselt number distributions

Figure 7 show the comparisons of the predicted results with the measured data for the wall shear stress at the hot wall reported in King [6]. The experimental data deviate severely from the symmetry. We observe that the original $\overline{vv}-f$ model predicts well the shear stress at the hot wall except near the upper wall region and predicts it poorly at the cold wall except near the lower wall region. The general trend of prediction of the wall shear stress by the modified $\overline{vv}-f$ model is the same as the original $\overline{vv}-f$ model and there is a small difference between predictions by two models. However, the prediction of the two-layer model is quite different from that by the $\overline{vv}-f$ models. We can observe that even the original $\overline{vv}-f$ model does not predict well the laminar to turbulent transition at hot wall observed in the experimental data. Figure 8 shows the comparisons of the predicted results with the measured data for the local Nusselt number at the hot wall reported in King [6]. The local Nussselt number given in Fig.8 is based on the temperature difference between hot and cold walls. Thus, some manipulations are made using the experimental data of centerline temperature given in Fig.4. As explained above, the measured data of the centerline temperature do not exhibit the linear variation due to insufficient insulation and this may affect the heat transfer coefficient. The original $\overline{vv}-f$ model predicts accurately the local Nusselt number at the hot wall except near the upper wall and the transition phenomenon at the lower portion of the hot wall is also well predicted. The prediction of the modified $\overline{vv}-f$ model is similar to that of the original $\overline{vv}-f$ model except that the modified model predicts a smooth transition. The two-layer model does not predict the transition

phenomenon. Kenjeres [5] under-predicts the local Nusselt number at the hot wall and the predicted transition is weak and delayed (y/H=0.26) when compared with the experimental data (y/H=0.1) and the present prediction by the original $\overline{vv}-f$ model (y/H=0.12).

5. Conclusions

The two $\overline{vv}-f$ models and the two-layer $k-\varepsilon$ model, both with algebraic heat flux model, are tested for natural convection in a rectangular cavity. The primary emphasis of the present study is placed on the evaluation of the $\overline{vv}-f$ models for the natural convection problem. The performances of turbulence models are investigated through comparison with available experimental data. The original $\overline{vv}-f$ model with AFM best predicts the mean velocity, velocity fluctuation, Reynolds shear stress, turbulent heat flux and the local Nusselt number and wall shear stress at the hot wall, and the predicted results agree fairly well with the measured data. In general the two-layer model predicts poorly all the variables when compared with the original $\overline{vv}-f$ model. We believe the length scales in the two-layer model based on the forced convection flows should be modified for the natural convection flows. The modified $\overline{vv}-f$ model predicts properly all the quantities, but there exist some differences between two $\overline{vv}-f$ models. The following conclusions are drawn from the present study;

(1) The two-layer model is numerically stable, however, it always under-predicts the turbulent quantities (Reynolds stresses, turbulent heat flux), thereby, the wall shear stress and the local Nusselt number.
(2) The original $\overline{vv}-f$ model (n=1) produces the most accurate solutions for all the quantities considered in the present study, particularly the local Nusselt number distribution at the hot wall. Within the present author's knowledge, the present prediction by the original $\overline{vv}-f$ model (n=1) is the most accurate solution among those reported in the literatures. However, this model exhibits the numerical stiffness problem in the segregate solution procedure like SIMPLE algorithm. A simple method proposed in the present study can be a way of avoiding such a numerical problem.
(3) The modified $\overline{vv}-f$ model (n=6) avoids the numerical stiffness problem and it predicts nearly the same level of accuracy and convergence compared with the original $\overline{vv}-f$ model although there exist some differences.

References

[1] Durbin, P. A "Separated flow computations with the $k-\varepsilon-\overline{v^2}$ model," *AIAA J.* **33**, (1995), pp. 659-664.
[2] Lien, F. S., Kalitzin, G. and Durbin, P. A., "RANS modeling for compressible and transitional flows", Center for Turbulence Research, Proceedings of the Summer Program, (1998).
[3] Chen, H. C. and Patel, V. C, "Near-wall turbulence models for complex flows including separation," *AIAA J.*, **26**, (1988), pp. 641-648.
[4] Kenjeres, S. and Hanjalic, K., Prediction of Turbulent Thermal Convection in Concentric and Eccentric Annuli, *Int. J. Heat Fluid Flow*, **16**, (1995), pp. 428-439.
[5] Kenjeres, S., Numerical Modelling of Complex Buoyancy-Driven Flows, Ph.D Thesis, Delft University of Technology, The Netherlands, (1998_
[6] King, K. V., "*Turbulent natural convection in rectangular air cavities*," Ph.D Thesis, Queen Mary College, University of London, UK. (1989).
[7] Cheesewright, R., King K. J. and Ziai, S, "Experimental data for the validation of computer codes for the prediction of two-dimensional buoyant cavity flows," *Proceeding of ASME Meeting*, HTD, **60**, (1986). pp. 75-86.